

## ADVANCED HIGHER MATHEMATICS

### SKETCHING RATIONAL FUNCTIONS 1

Sketch the graph of each rational function, showing clearly all points of intersection with the coordinate axes and asymptotes. **You need not find the coordinates of any stationary points.**

1.  $y = \frac{1}{x-1}$

2.  $y = \frac{1}{x+2}$

3.  $y = \frac{4}{2x-1}$

4.  $y = \frac{6}{x-2}$

5.  $y = \frac{1}{(x-2)(x-4)}$

6.  $y = \frac{1}{(x-1)(x+3)}$

7.  $y = \frac{1}{x(x-1)}$

8.  $y = \frac{1}{x^2-4}$

9.  $y = \frac{1}{x^2-x-2}$

10.  $y = \frac{1}{x^2+3x-4}$

11.  $y = \frac{x}{(x-1)(x-3)}$

12.  $y = \frac{x-4}{(x+1)(x-2)}$

13.  $y = \frac{x-1}{x(x+1)}$

14.  $y = \frac{x+3}{(x-1)(x+2)}$

15.  $y = \frac{x-2}{(x-4)(x+1)}$

16.  $y = \frac{x-1}{x^2}$

17.  $y = \frac{x}{(x+2)(x-3)}$

18.  $y = \frac{x+1}{(x-1)^2}$

19.  $y = \frac{x-1}{(x-2)(x+1)}$

20.  $y = \frac{x}{x^2-1}$

21.  $y = \frac{x-4}{x^2-4}$

22.  $y = \frac{x-1}{x^2+x-6}$

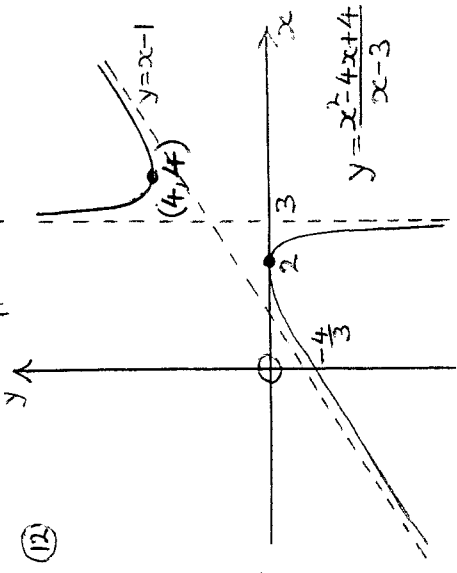
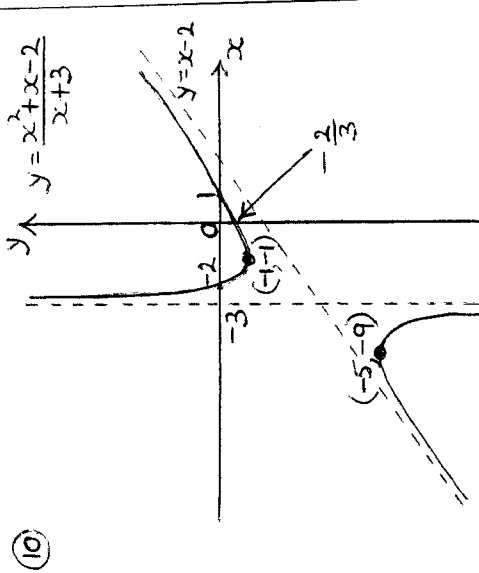
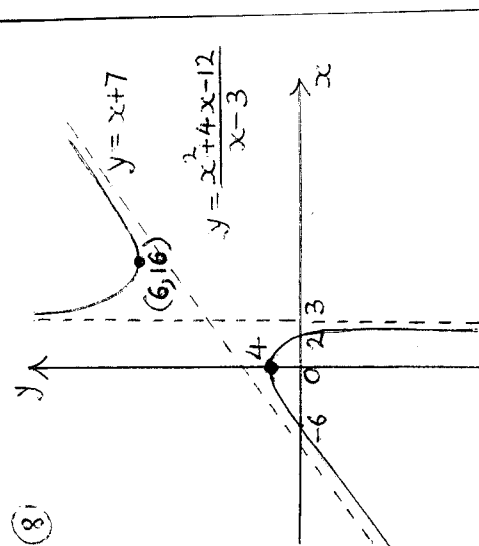
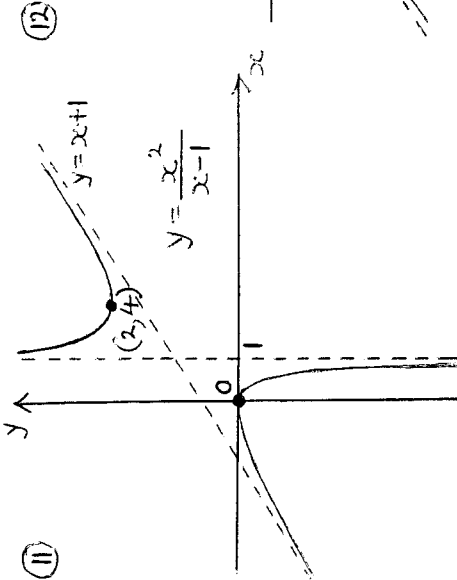
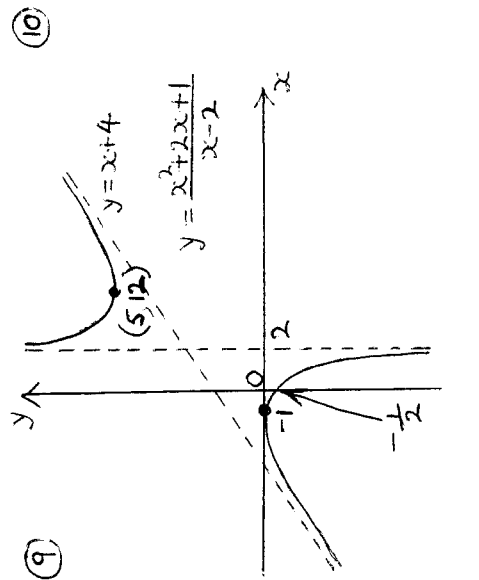
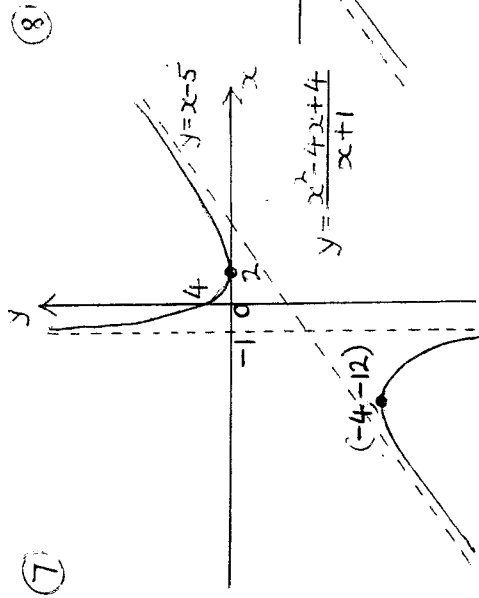
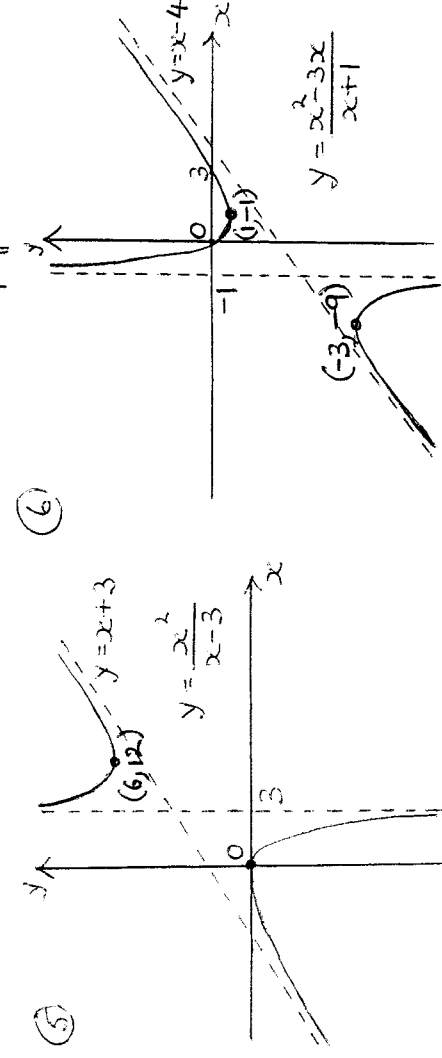
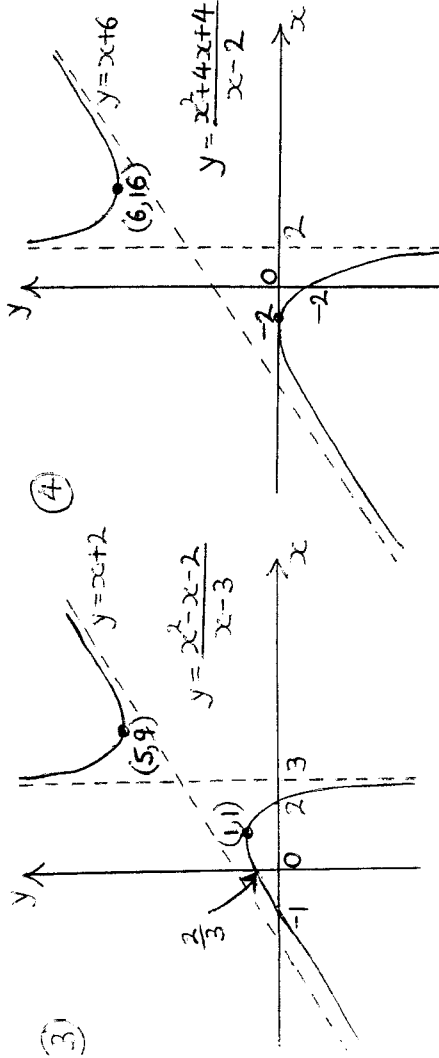
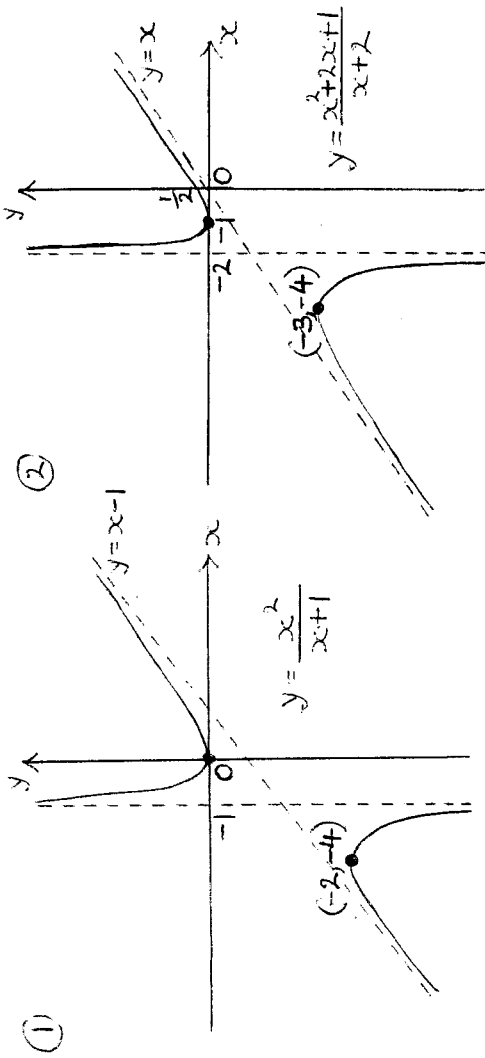
23.  $y = \frac{12}{x^2-2x-3}$

24.  $y = \frac{x+3}{x^2-1}$

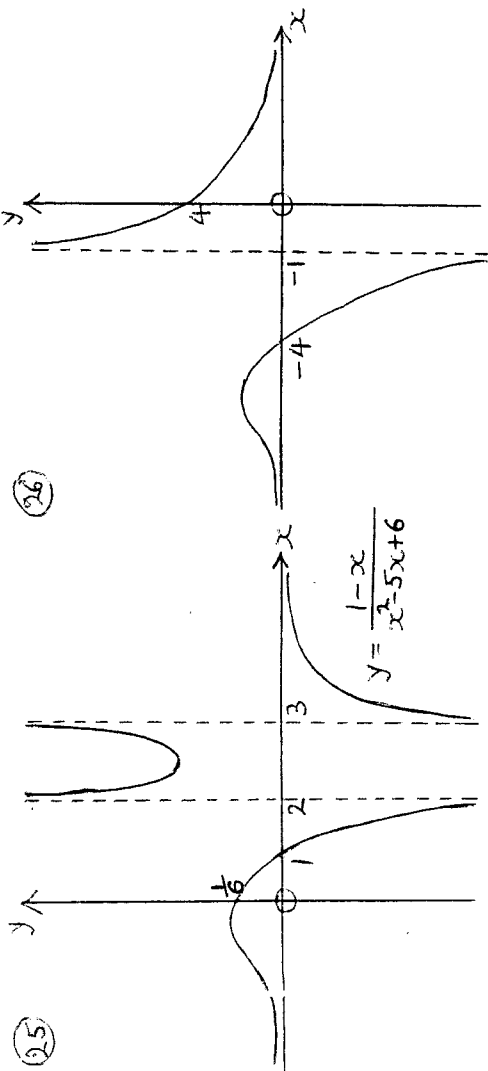
25.  $y = \frac{1-x}{x^2-5x+6}$

26.  $y = \frac{x+4}{x^3+1}$

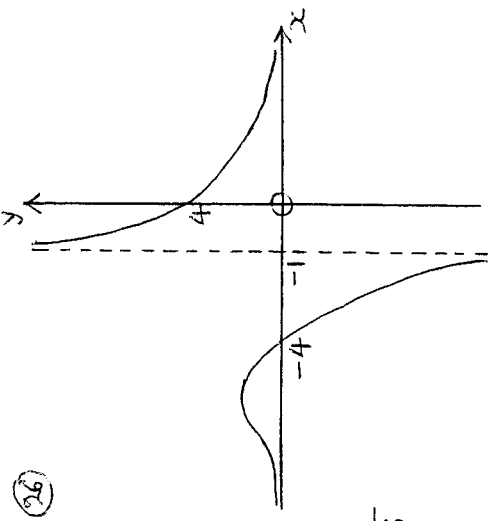
## ANSWERS



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**SKETCHING RATIONAL FUNCTIONS 3**

1. The function  $f$  is defined by

$$f(x) = \frac{x^2}{x+1}, \quad x \neq -1.$$

- Find the coordinates of all the points of intersection of the graph of  $y = f(x)$  with the coordinate axes.
- Find the equation of each of the two asymptotes on the graph of  $y = f(x)$ .
- Find the coordinates of each of the two stationary points on the graph of  $y = f(x)$  and determine their nature.
- Sketch the graph of  $y = f(x)$ , showing clearly all relevant features.
- State the range of values of the constant  $k$  such that the equation  $f(x) = k$  has no real solutions for  $x$ .

**For each of the functions defined in questions 2 to 20, repeat all the parts of question 1.**

- $f(x) = \frac{x^2 + 2x + 1}{x + 2}$
- $f(x) = \frac{x^2 - x - 2}{x - 3}$
- $f(x) = \frac{x^2 - 4x + 4}{x + 1}$
- $f(x) = \frac{x^2 - 4x + 4}{x - 2}$
- $f(x) = \frac{x^2 - 3x}{x + 1}$
- $f(x) = \frac{x^2 + 2x + 1}{x - 2}$
- $f(x) = \frac{x^2 - 4x + 4}{x - 3}$
- $f(x) = \frac{x^2 - 4x + 4}{x - 1}$
- $f(x) = \frac{x^2}{x + 2}$
- $f(x) = \frac{x^2 - 5x + 4}{x}$
- $f(x) = \frac{x^2}{x + 3}$
- $f(x) = \frac{x^2 + 4x + 4}{x - 3}$
- $f(x) = \frac{x^2 - 2x + 1}{x - 3}$
- $f(x) = \frac{x^2 + 4x + 4}{x - 3}$
- $f(x) = \frac{x^2 - 2x + 1}{x - 3}$

21. The function  $f$  is defined by

$$f(x) = \frac{x^2 + 2x + 5}{x + 1}, \quad x \neq -1.$$

- Find the coordinates of the point of intersection of the graph of  $y = f(x)$  with the  $y$ -axis.
- Show algebraically that the graph of  $y = f(x)$  does not cross the  $x$ -axis.
- Find the equation of each of the two asymptotes on the graph of  $y = f(x)$ .
- Find the coordinates of each of the two stationary points on the graph of  $y = f(x)$  and determine their nature.
- Sketch the graph of  $y = f(x)$ , showing clearly all relevant features.
- State the range of values of the constant  $k$  such that the equation  $f(x) = k$  has no real solutions for  $x$ .

## ADVANCED HIGHER MATHEMATICS

### SKETCHING RATIONAL FUNCTIONS 2

Sketch the graph of each rational function, showing clearly all points of intersection with the coordinate axes and asymptotes. **You need not find the coordinates of any stationary points.**

1.  $y = \frac{x+1}{x}$

2.  $y = \frac{x}{x-1}$

3.  $y = \frac{x+1}{x+2}$

4.  $y = \frac{x+2}{x-2}$

5.  $y = \frac{x-1}{x+1}$

6.  $y = \frac{2x+1}{x-1}$

7.  $y = \frac{x+1}{x+4}$

8.  $y = \frac{x^2+2x}{x+1}$

9.  $y = \frac{x^2}{x+1}$

10.  $y = \frac{x^2-1}{x+2}$

11.  $y = \frac{x^2}{x-2}$

12.  $y = \frac{x^2-4}{x+1}$

13.  $y = \frac{x^2-2x}{x+1}$

14.  $y = \frac{x^2}{x+2}$

15.  $y = \frac{x^2-4x+3}{x-2}$

16.  $y = \frac{x^2+x-2}{x-2}$

17.  $y = \frac{x^2-3x-10}{x-2}$

18.  $y = \frac{x^2+2x-3}{x+2}$

19.  $y = \frac{x^2+1}{x-1}$

20.  $y = \frac{x^2}{x^2-4}$

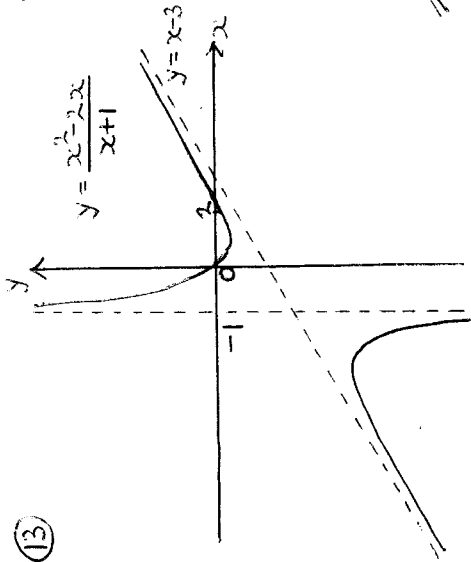
21.  $y = \frac{x^2+x}{x^2+x-2}$

22.  $y = \frac{2x^2}{x^2-1}$

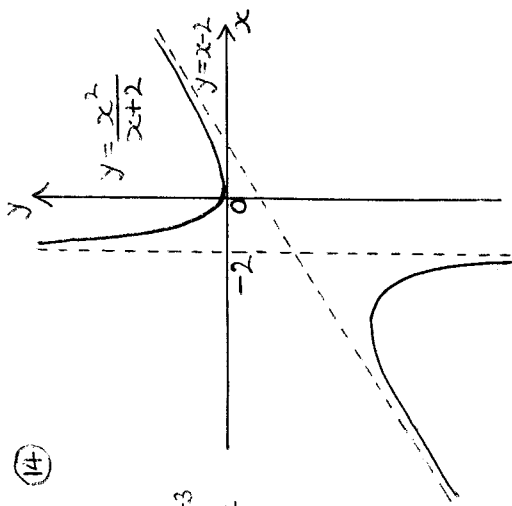
23.  $y = \frac{x^2+1}{x^2-1}$

24.  $y = \frac{x^2+2x-8}{x^2+x-2}$

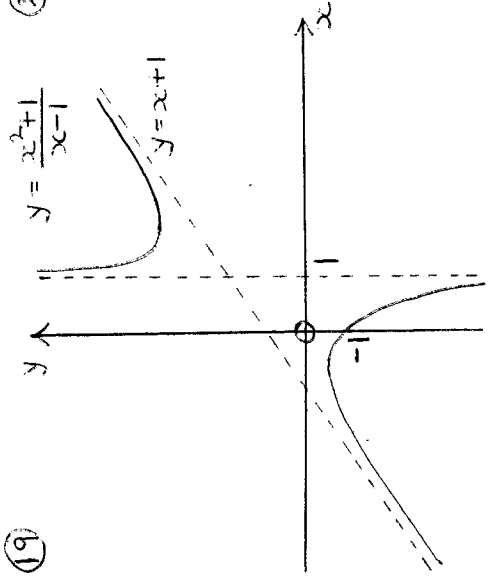
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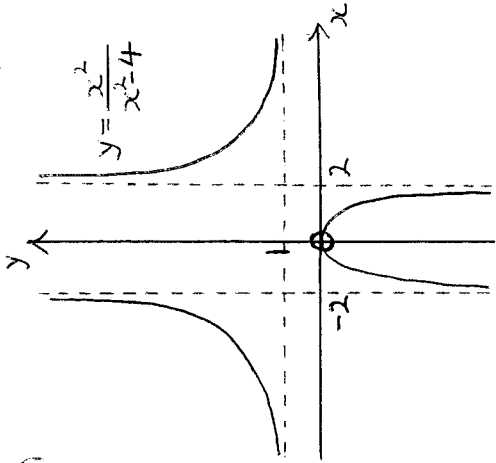
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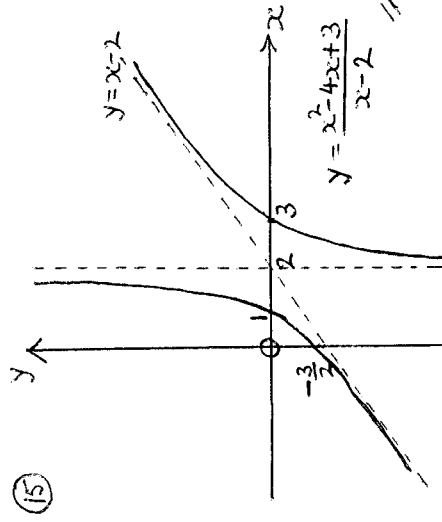
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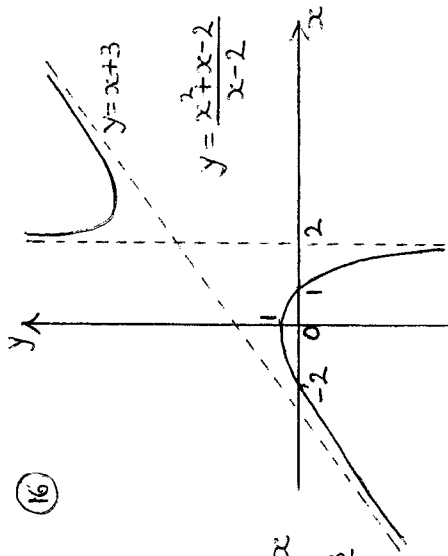
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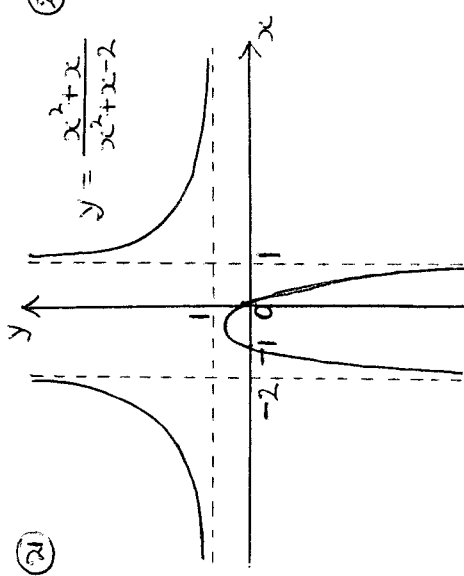
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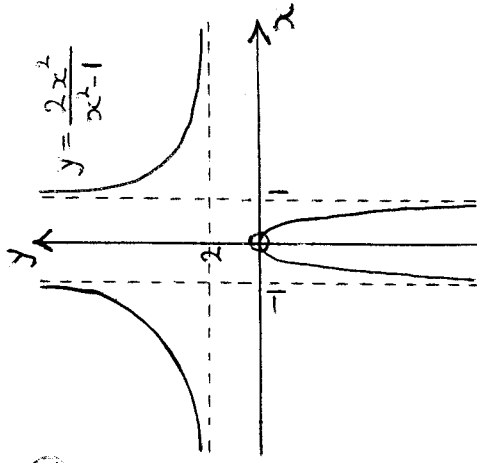
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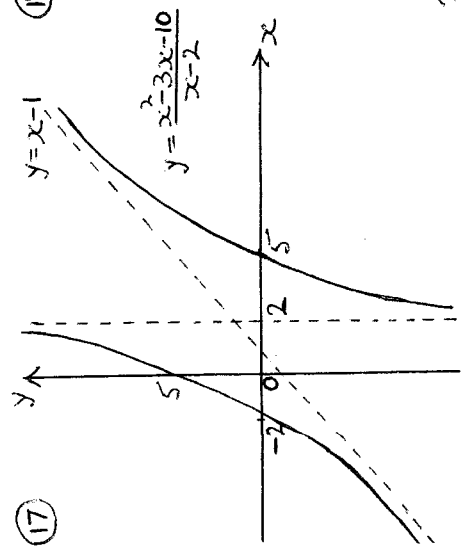
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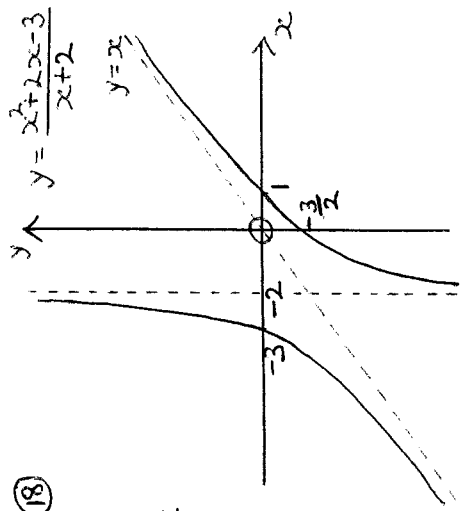
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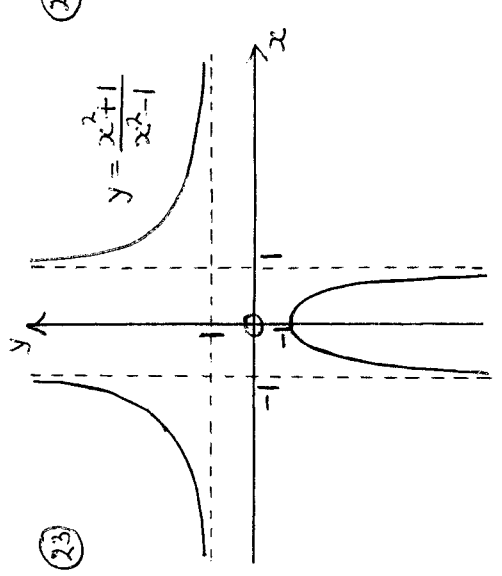
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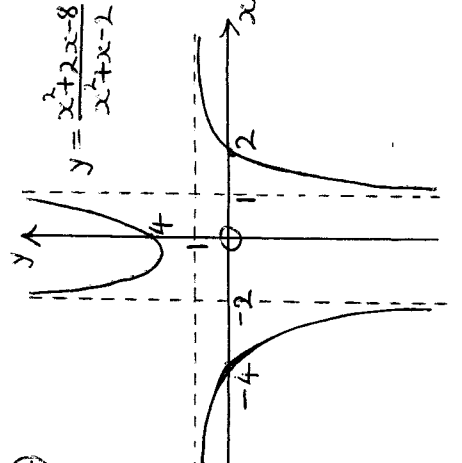
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# ANSWERS

